NEWTONIAN DYNAMICS REVISITED

This book is about Newtonian mechanics from the perspective of the laws of motion – the Law of Inertia or Newtons first law of motion, Newtons second law of motion primarily, and touching a bit on Newtons third law of motion too, pertaining to action and reaction forces.

-by Retd. Professor SURESH ROBERT NORMAN

Chapter 1 covers ground relevant to the first two laws of motion principally, with a brief historical background of the development of ideas and concepts of inertia and motion from the times of ancient Greece and the philosophical quests of Aristotle to Galileos bold scientific ideas and onwards to Newtons elucidations of the Physics of inertia and motion as enshrined in his laws of motion. Also included are a few fresh perceptions and ideas developed by me that may seem unconventional at first sight, but would become clearer as you navigate through this book. For example models of inertia that illustrate Inertial resisting force diagrammatically and mathematically, along with descriptions of these models.

Chapter 2 is essentially a study of a one dimensional collision between two equal spherical masses, though approached in quite a different way which may interest some readers I think. The gist of this chapter is exploring energy as the prime mover and its direct links and chain of action resulting in motion; or the natural flow from energy to conventional mechanical force, and via the force agency or physical force action to change in motion – in other words, the mantra ' energy is the fuel, force the arm, and motion is the result '

The collision is analysed mathematically starting from energy input to force generation to motion actualization in that sequence. Also, a reverse engineering exercise that produces a very good fit of energy to force and into Newtons second law of motion and its equations viz. Force = mass x acceleration or F = m x a and Ft = mv or Fdt = mdv (depending on whether the acceleration under consideration is steady, average or an instantaneous acceleration).

Chapter 3 again focusses on energy and the implications of the magnitude of energy input and the proportional amount of motion it produces and how energy can be viewed as time delay. Since a very large energy input implies a higher kinetic energy imparted to the body, and more acceleration and a faster rate of covering distance. That is, it is tantamount to less time delay in the propelled body reaching its destination from its starting point in space.

However, it is assumed the velocity of the body is much less than c, the speed of light. Hence Einsteins Relativistic effects are not considered here, only a purely Newtonian dynamics scenario.

Chapter 4 deals mostly with higher orders of equations of motion, considering jerks and yanks and higher orders of differential rates of forces and their accelerations. This subject has been dealt extensively by specialists as can be seen in the Article references given.

1) Note on Inertia and Motion :

https://suresh-norman.angelfire.com/NOTE_on_INERTIA_and_MOTION.pdf

2) An additional study note on Newtonian Mechanics. This note on one dimensional collision dynamics is enclosed below. The other Chapter notes are given as clickable links.

3) Note on Energy, Space and Time done by me in 2020 August - October.

https://suresh-norman.angelfire.com/Energy__Space__Time.pdf

4) Note on Physics of a Mechanics topic:

https://suresh-norman.angelfire.com/Physics_of__yanks.pdf

An additional study note on Newtonian Mechanics – by Prof. Retd. Suresh Robert Norman

Introduction

' Energy is the fuel, Force is the arm, and Motion the result '

My earlier notes pertained to Newtonian dynamics, Newtons laws of motion and in particular the first two laws of motion viz. The law of Inertia and Newtons 2nd law of motion.

This note is primarily a study of force and the motion it causes, but from the energy perspective. In short, an exercise in reverse engineering ! Which takes one closer to the roots of the second law of motion.

I begin with the premise that Energy is the prime mover and force is the arm of energy by which motion and action results.

And a brief look at the springs of pure knowledge to derive the significance of the ubiquitous chain of pure energy - force - motion.

[1] Consider a one dimensional collision between 2 balls. Each ball is identical to the other and both have the same mass 'm', and m= 1Kg. The first ball, ball1, is moving with a velocity ' v' m/s, and is heading for a collision with ball2, which is stationary with velocity v = 0 m/s.

The ball2 which is an obstacle in the path of ball1, would naturally alter the state of motion of ball1.

In the process of the collision and its impact, there are energy changes, force changes, and velocity and momentum changes in both the bodies. We know that the collision acts as an energy- force Exchange, where the distribution of energy, force and momentum in the colliding bodies depends on their relative masses, the initial velocities of the two bodies and the time duration of the collision.

[2] Now, a bit of reverse engineering for the purpose of study.

The energy Source here is ball1, which is assumed to be moving at a constant velocity 'v' and possesses a kinetic energy $E = ½mv^2$.

During each instant of the collision time, the velocity of ball1 would decrease progressively, since it is forced to decelerate by the obstacle in its path which is ball2.i

At the start of the time duration of collision, at t = 0 second, ball1 has velocity 'v' m/s and energy $E = ½mv^2$.

For every ' dv ' decrease in the velocity of the striking ball, ball1, there is a proportional but non-linear decrease 'dE ' in its energy as per the energy equation, $E = ½mv^2$.

Consider two velocity data points of ball1, v and v – dv , at the instants t = 0 and the next instant t = t + dt.

The corresponding energy values are

$E = ½mv^2$ and

$(E – dE) = ½m(v - dv)^2$

Hence,

$(E - dE)- E = (½mv^2 - mvdv + ½mdv^2)- ½mv^2$

$= -mvdv + ½mdv^2$

$= -mvdv$, neglecting dv^2 term since ~ 0

Hence $(E – dE) – E = - dE$ is the energy differential change corresponding to the velocity differential change – dv , from

$(v – dv) – v = - dv$.

And $-dE = -mvdv$

Hence the ratio of change of dE and dv is (-dE/-dv)

= dE/dv = mv (ie. instantaneous momentum value of ball1 over a dv interval. Both are vector quantities, so the (dE/dv) vector corresponds to the momentum vector)

For each such equal dv variations of velocity of ball1, there will be corresponding non-linearly varying values of dE. This is because the data points v1, v2, v3, ...and energy values E1, E2, E3, ... at different instants since the collision, are related non-linearly in the energy equation

$E = ½mv^2$.

Hence (dE/dv) varies nonlinearly as velocity decreases incrementally with each decrementing instant of time, from the start of the collision time till the entire time duration of the collision.

Also by differentiating the energy equation $E = \frac{1}{2}mv^2$, we know $dE/dv = mv$.

ie. dE/dv has larger values when v has higher values, and E varies non-linearly with v , as also explained earlier and has much higher values at higher v, since E propnl. to v^2. Also velocity and dE/dv are varying with time.

Hence the energy exchange from ball1 to ball2, when it rams into stationary ball2 is proportionately more at the start of the collision and tapers non-linearly or exponentially from the start of the collision to the middle and till the end of the collision time interval.

Hence, during the collision,for equal infinitesimal intervals of time dt, the time variation of dE/dv changes will also be non-linear. ie. of $d(dE/dv)/dt$

Since $dE/dv = mv$,

$$d(dE/dv)/dt = d/dt\,(mv)$$

$$= m(dv/dt) + v(dm/dt)$$

$$= m(dv/dt)$$

(since we neglect the $v(dm/dt)$ term, because $v \ll c$, the speed of light , so Einsteins Relativistic mass changes do not apply here, and only Newtonian laws of motion). Hence

$$d(dE/dv)/dt = m(dv/dt)$$

= mass x acceleration = ma = F, Force

ie. The decelerating ball1 is exerting or producing a ' mass x acceleration' driving force pressing against ball2. And this force rises non-linearly or exponentially, corresponding to and in proportion to the nonlinear decrementing amounts of dE/dv values with increasing time increments in the collision time. The incremental force vectors dF vary according to the time rate of change of the (dE/dv) vectors, ie. the instantaneous momentum vectors mv and they nearly mirror the time rate of change of output (dE/dv) vectors or the output instantaneous momentum vectors.

[Also $(dE^2 + dv^2)$^0.5 = dF approx.

$$1 + (dE/dv)^2 = (dF/dv)^2$$

ie. $\pm dE/dv \sim \pm dF/dv$

(ie. incremental energy values, dE and incremental force dF values are closely matched for each dv incremental change)

Also, since $dE/dv = mv$,

$$1 + (mv)^2 = (dF/dv)^2$$

ie. $\pm mv \sim \pm dF/dv$

On integrating, $\frac{1}{2}mv^2 \sim F$!

(valuewise, though the dimensionality does not look quite right at first sight.

Considering collision time t=1sec., v =1m/s, & $mv/t = F$,

$mv/(2s/v) = F$, & s = avg. velocity x t =

$v/2 \times t$, ie. $\frac{1}{2}mv^2 = F s = F$, since s = 1m.)]

The force F varies each instant from the start to the end of the collision, and we may conveniently consider the nonlinear force variations of F during infinitesimal time intervals dt, during the collision time interval.

Since this is not a static force but a dynamic force, it acts dynamically, transferring the striking ball1s energy and momentum to ball2 in a piecemeal manner with every dt time interval. Hence the cumulative energy and momentum of ball2 after the collision bears a good correspondence with the energy and momentum of ball1 before the collision (neglecting the energy losses).

Since,

$d(dE/dv)/dt = m(dv/dt) = ma = F$, hence

$mdv = Fdt$

integrating over the collision interval, 0 to t, the net Impulse force action is tantamount to grabbing the momentum and energy of the striking ball1 and passing it progressively and incrementally in a proportionately nonlinear manner to ball2.

[Remember earlier, from ball1,

it's nonlinearly changing rate of $dE/dv = mv$, at each of its particular velocity values v, and its corresponding changing time rate

$d(dE/dv)/dt = d/dt\,(mv)$

$\quad\quad\quad\quad = m(dv/dt)$

$d(dE/dv)/dt = m(dv/dt)$]

Also,

$mdv = Fdt$,

as above, replicates. ie. from ball1 to ball2

$mdv = Fdt = mdv$, with the Force mechanism acting as the energy exchanger in the collision process !

Input E, being differentiated during the collision process time, yields a proportional nonlinear force dF at each stage of differentiation, and eventually the summed dF = F. In parallel, the dFs are being propagated symmetrically to the ball2 as its output momentum and kinetic energy mirrored from the input end kinetic energy of ball1.

$E = \frac{1}{2}mv^2 \rightarrow dE/dv = mv \rightarrow d(dE/dv)/dt = m(dv/dt) \rightarrow ma \rightarrow F$ &

$mdv = Fdt \rightarrow mdv \rightarrow dE/dv \rightarrow \frac{1}{2}mv^2 \rightarrow E$

Output E.

Concluding experimental analytic approach

Also, consider a typical model exponentially rising graph of a force function F^* with v, with the value of the force varying from 0 to F, the force functions final value at t= 1 sec

.

Now we can consider small increments of dv along the x-axis, from v=0, to v= v m/s (the final velocity of ball2), with their associated dF increments.

Also, each of the dF variations in each dv interval are associated with a particular or corresponding dE Energy change. That is to say, each dE/mv = dv , and each dE change is tied to its corresponding dF change.

Then,

$$\int_0^F F^*dv$$

$$=\int_0^F F.dv \sim= \sum_0^F \sum_0^{dv} \left(\tfrac{1}{2}\, dF.dv\right)$$

$$\text{Ie. } \int_0^F F.dv \sim= \sum_0^F \sum_0^E \left(\tfrac{1}{2}\, dF(dE/mv)\right)$$

(Average dF change in each dv interval= ½dF,

And using F as the resultant vector sum of all the dF vectors and dE/mv= dv)

$$F \times v \sim= \tfrac{1}{2} F \times (E/mv)$$

$$F \times v \sim= \tfrac{1}{2}\, mv/t \times E/mv$$

$$Fv = \tfrac{1}{2}\, E/t$$

The variations of dv with dt (or incremental accelerations da) are associated with corresponding dF and dE changes . Also, the dt intervals are taken constant , unlike the varying dv intervals, to accommodate changing incremental accelerations da in the same x axis. More to be imagined than portrayed !

[In the next section where I interpreted or offered my explanation,

the fast rise in 0.1 second of the Force to F = 0.5 N from 0 N, corresponds to the slow rise of the velocity of ball2 from 0 m/s to 0.5 m/s in 0.5s, (and this happens in parallel as ball1 decreases in velocity from 1 m/s till 0.447 m/s), in half the collision time or 0.5 second.

Thus in the equation $Fv = ½ E/t$,

$0.5 \text{ N} \times 0.5 \text{ m/s} = ½ \times 0.5 \text{ J}/ 1 \text{ s}$, or

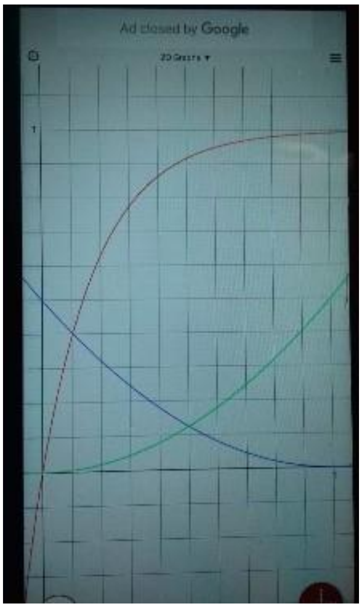
$0.25 \text{ N/m/s} = 0.25 \text{ J/ s}$

proving the integrity of the equations and this approach to understanding the collision process]

$Fv = ½ E/t$,

Implies the average rate of change of the Input Energy E, with respect to time , or the average Input power is being converted to work output with the exponentially growing action force F at the rate of the output velocity being delivered.

For the experimental graph shown - this is my interpretation or explanation.



I have used average approximations not the exact values of Force and velocity.

The blue graph shows the kinetic energy E of ball1 decreasing from 0.5 J to 0 J. Here,

0.5 J corresponds to velocity 1.0m/s of ball1 at time t=0 second and when velocity of ball1 is 0m/s, it's kinetic energy E is 0J .

The red graph shows the nonlinear exponential rise of the Action Force from 0 Newton at t=0 sec. to 1 Newton at time t= 1.0 sec.

The green graph shows the kinetic energy E of ball2 rising from 0J at t=0sec. to 0.5 J at t= 1 second, when it reaches the velocity of 1m/s.

These graphs model fairly closely the Collision system dynamics and the mathematical equations of the study earlier.

It can be seen that ball1 velocity v changes from 1m/s at t=0 sec. to 0.894 m/s at the end of the first 0.1 second of the collision time. Thus dv is ~ 0.106 m/s during the first 0.1 second time interval.

(check in the graph, 0.5J = $\frac{1}{2}mv^2$, v = 1 m/s and 0.4J = $\frac{1}{2}mv^2$ gives v = 0.894 m/s.)

Using the approximation dv =0.106 m/s,

dE = mvdv = 1Kg. x 1m/s x 0.106m/s =0.106 J (the value in the graph is 0.1 J)

So dE/dv = mv = 0.106 J/0.106 m/s =1.0 J/ m/s = 1 Ns.The d/dt(dE/dv) change of 1.0 Ns in a time interval 0.1 s, is 0.1Ns.

During the ball1 (dE/dv) vector change in dt = 0.1 sec., avg.force F builtup on ball2 or dF vector changes from 0 N to 0.5 N but in 0.5 second.

(The actual force difference values can be calculated or picked up from the Graph in the graph Plotter App. I chose to take the nearest possible values from the Graph directly without computing them. So they may be just near the real values.

[

For those who want to compute the values, the function I used is

F* = (1- e^ (-t/0.15s)

]

The values from the Graph Plotter graph, from t=0 till the end of the first interval, ie. after 0.1 second, equals ~ 0.489N. That is a delta F ~ 0.489N.

Similarly for subsequent intervals, (before the turning point considered as the midpoint of the collision time, ie. 0.5 second), the delta F values are, 0.2479 N, 0.1291N, 0.066N, 0.034N.

This totals to 0.489N + 0.477 N = 0.966 N, within the first half of the collision.

Or an average, force of 0.2N per 0.1 sec. time interval in the 0.5 sec first half of the collision.

As shown above the total force of 0.966N in the first half of the collision is almost 1N, or the overall Force applied. This gives an acceleration of 1m/s/s, during the 0.5 second period, hence after 0.5sec., the ball2 runs away with a velocity of 0.5 m/s.

Also, we can directly observe the computed values of dF/dt that appear on the graph (thanks to Philip Steven's graph Plotter App !). This obviates manual computing of dF/ dt values.

Knowing the corresponding Tan theta values from these dF/dt values, we can get theta values and then compute the Cosines of these theta values.

These can be used to estimate the corresponding projected values of the Force vectors in each 0.1 second interval.

In the first 0.1 second interval, the dF/ dt value is 3.4 and the force value is 0..489 N. The corresponding angle is the inverse Tangent of 3.4 = 73.6 °.

Hence the projection of the force vector is

0.489N x Cos 73.6 ° = 0.14 N.

Similarly in four of the following 0.1 second intervals, the projection of force vectors are,

0.2479N x Cos 60.3° = 0.1064 N.

0.1291N x Cos 41.8 ° = 0.049N.

0.066N x Cos 24.6 ° = 0.6N

0.034N x Cos 13.3 ° = 0.033N.

The total of the projected values of the Force vectors along the x axis which is the time axis , or their divergence dot products

= 0.37 N.

Maybe I need to check again and see.

Also, if one concatenated all such force vectors you would get a resultant vector at an angle of 45 ° to the +ve x axis with a magnitude=√2N. Projection of resultant vector= √2x 1/√2 Ns= 1Ns.).

From the Fdt = mdv equation, the Force change of 0.5N should have produced a change in velocity of ball2 of 0.5m/s , which can be verified from the green graph, showing the growth in KE of ball2. Kinetic Energy of ball2 is ½mv^2 = 0.125 J. This dv of 0.5m/s is done in dt of 0.5 second. That is, the average change in velocity of ball2 in dt of 0.1 sec. = 0.1 m/s. Thus the average acceleration of ball2 is 1.0 m/s/s during this 0.5 second time interval. Similarly in the second half of the collision. So the average force coming into play in both halves of the collision is equal to a force of 1 Newton.

(from the well known and most universally used equation on earth and space !

force = mass x acceleration

or, Newtons second law of motion)

Though the instant by instant force graph during the collision manifests as a curve having bumps and nonlinearities pervading it, which at best can be piece-wise linearized for analysis.

(Actually the Force function needs to be modified slightly from the in the graph I used. The change in Force dF in dt of 0.1 sec is F=0.4 N and not 0.5N as it should be as per calculation.

In the second iteration I modified the Force equation, and this tallied perfectly with the expected calculations.

In the first experiment I used an estimated mechanical time constant of 0.2 second, which is fairly close to the exact value I got experimentally as shown on the second picture uploaded. It is 1.333333333333 times more than the better fitting time constant !

Here, in the second experiment the time constant is 0.15 second.

Thus, through the collision time of 1 second, there are approx. 6.6666666666666 time constants !!

Though in actual physical experiments carried out in the laboratory, with the second ball, ball2, replaced by a mechanical arrangement of a handglove attached to a spring, much higher time constants are determined eg. of the order of 1 second.

The interesting time-constant I recently accidentally found to be very interesting is a time constant of 0.005 second or 5 milliseconds.

With such a low order of time constant, the exponential curve is very much linearized and fast rising.

Also, the product of change in Force and time (or the Force Impulse) in each tenth interval of the collision is consistently very near to a force of 0.1 Newton-second, from the first time interval of 0.1 sec. to the last time interval of 0.1 sec , upto the end of the collision . And the sum of Force Impulse values corresponding to each one tenth time interval of the collision on an average, nearly equals 1Newton-second, as it should be.

That is approx. 1Newton force has been acting over a second. Which gives an acceleration of very nearly 1 m/s/s, uniformly or consistently across the collision time of 1 second.

This also points to minimum energy loss in the efficient transfer of input energy of ball1 to the output energy of ball2.

Reminds me of the efficient transfer of energy in the swinging balls in Newtons cradle.)

Also from the equation $KE = ½mv^2$, you can confirm that the change in velocity of ball2 is dv = 0.5m/s in 0.5s dt, as seen in the graph. From the Graph, there is a change of 0.025 m/s in the first 0.1 second, with ball1 change of 0.1 m/s during the first 0.1 second .

This corresponds to $dE = ½ \times mv^2 = 0.5 \times 1Kg \times 0.1^2 = 0.005$ J

dE/dv = 0.005J/ 0.025m/s = 0.2 J/m/s or 0.2 Ns.

For dt = 0.1 sec., d/dt of dE/dv=0.2 Ns is 0.02 Ns. So the force on ball2 in 0.1sec.= 0.02 N.

This rate of vector change is exactly 1/5 th of that for ball1 during first 0.1sec.

An early correspondence is thus established between initial decreasing change in KE of ball1, force F rise, and the increase in output KE of the ball2 ...

The graphs shown are taken from the Graph Plotter mobile App.

References –

1) Physics, Teachers Edition, Irwin Genzer, Philip Younger.

2) The Feynman Lectures on Physics Vol 1.

3) Elementary Textbook on Physics – Edited by G.S. Landsberg, MIR Publishers

4) Mr. C.D.Norman , former Physics Teacher, Bombay Scottish High school. He was my Physics teacher at school and also discussed Physics topics with me including collisions and momentum transfer even in his mid-nineties ! Probably the main inspiration behind this note and my earlier Physics notes.
