**NOTE on INERTIA and MOTION**

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‘ **Nothing in Nature is more ancient than motion , and the volumes that the philosophers have compiled about it are neither few or small ; yet have I discovered that there are many things of interest about it that have hitherto been unperceived** ‘ – Galileo Galilei

**Introduction**

The beginning of most things requires sufficient or more force , in order to overcome rest Inertia. Even God, had to arise from his primeval state of rest and slumber before kick-starting into motion the Universe !

Inertia is why we have to exert more force when sitting up from a reclining position , or standing upright, or starting to walk or run ( also for a humanoid robot ) . To overcome the inertial reaction or resisting force faster , more force needs to be applied on a body. And the more bulkier the body the more the force required to get it moving from rest.

Also when starting a bike , more throttle is applied initially to overcome faster this inertial resisting force before the desired acceleration and velocity is obtained with the related throttle. Electric and diesel locomotives also provide the high starting torque needed to overcome the inertia of rest of their heavy payloads ( In addition to inertial resisting forces , air drag , contact friction and gravitational forces also come into play in the physical world ).

In this Article , the author revisits the foundations of inertia and motion , and proposes a new theoretical framework to understand the mechanisms and working of inertial resisting or reaction forces. Since not enough literature exists in this area , the author has referred to the earlier ideas of inertia and motion established by Galileo and Newton . The study of inertial reaction forces is an analytical one and supported by my graphical and quantitative models and their self-contained equations.

**[ A ] Introduction to Motion and Inertia**

In todays’ modern world with its advanced mechanical science , technology & engineering and mechatronics and robotics, delving into the basic foundational elements of motion and rest may seem anachronistic and archaic. However, I hope the reader who studies this article is benefitted thereby.

Galileo was one of the first to point out that **the motion of a body , once started is not caused but simply exists** ( refer figure 1 )

When Aristotle was trying to study ‘ quantity of matter ‘ and ‘ quantity of motion ‘ , he was asking universal questions such as “ **What is the cause of motion ?**” or “ **Why motion exists ?**” and “ **Why do heavier objects fall faster ?** ”

However , he attempted to answer these questions purely by philosophic enquiry and reasoning but not backed by scientific experiments , and tried to answer these questions as per the prevalent ideas existing then.

Galileo’s genius approached the problem of motion with his question “ **What causes a change in motion**?” and thus he could find the answer to the ancient question “ **What causes motion ?** ”

He said it is “natural” for objects to stay in continuous motion , and that a force is not required to maintain this motion , which is maintained by its inertial mass as its momentum (No force, no change in motion ). He chose to observe and study motion by elegant and creative scientific experiments in order to gain a sound understanding of motion and better describe motion in order to discover “ What causes a change in motion ?”

He said you don’t have to do anything to keep a body moving ( ie. no force is required ) – only stopping it requires a force, to change its present state of motion and bring it to rest.

Galileo suggested that if a body is in motion , it will stay in motion along a straight - line path unless there is a push or pull force that causes it to move otherwise - “ **Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed**.”

**Galileo’s hypothesis became the Law of Inertia** which is all about ‘ no change in motion’. *This property of matter , of bodies persisting in their state of motion is called inertia.*

|  |  |
| --- | --- |
| No forces act on ‘m’ in this window     |  | | --- | |  |   velocity=’v’    body mass = ‘m’ |
| An obstacle is introduced in the window  velocity=’v’      body mass = ‘m’   |  | | --- | |  | |

**Galileo’s viewing window** – push and pull forces exist only outside this window.

The body ‘m’ has velocity ‘v’ and moves in a straight line with no force acting on it or required to sustain its motion. The state of motion of the body is maintained , also its kinetic energy ( Leibnitz’s ‘vis viva’) and its momentum ( mv or mass x velocity ).

For example - inter-stellar matter (or a man-made object like Voyager) hurtling through the near-vacuum of the vast expanse of space, presently free of any gravitational tugs(zero space-time curvature).

Also , in Galileo’s window , we are not interested in the previous history of forces that acted on the body and imparted to it its present state of motion.

However, this same body becomes a force agent only when an obstacle body in its path attempts to bring it to a stop , or if it collides with another body that intersects its path.

**Fig. 1** Inside **Galileo’s viewing window**

In the real world, surface contact frictional forces, air damping forces and other interfering forces may not make such an ideal window a practical possibility.

**Newton defined the Law of Inertia**  thus -

**“ The *vis insita* , or innate force of matter , is a power of resisting , by which every body , as much as in it lies , endeavours to persevere in its present state , whether it be of rest , or of moving uniformly forward in a straight line.**

**This force is ever proportional to the body whose force it is ; and differs nothing from the inactivity of the mass , but in our manner of conceiving it. A body , because of the inactivity of matter,is not without difficulty put out of its state of rest or motion. Upon which account , this vis insita , may by a most significant name , be called vis inertiae , or force of inactivity. But a body exerts this force only , when another force , impressed upon it , endeavours to change its condition , and the exercise of this force may be considered both as resistance and impulse ; it is resistance in so far as the body ,for maintaining its present state , withstands the force impressed;**

**It is impulse , in so far as the body , by not easily giving way to the impressed force of another , endeavours to change the state of that other. Resistance is usually ascribed to bodies at rest , and impulse to those in motion;………..”**

That is to say , the vis inertiae , or the inertia of a body strives to maintain its state of rest - opposing any impressed force that seeks to dislodge it from this state of rest , and when in motion it opposes any force that seeks to alter its course of motion to another state of motion or rest .The high school physics definition of the Law of inertia or Newton’s first Law of motion is far simpler , though it contains or embodies the essence of these ideas stated by Galileo and Newton – *A body at rest continues to be at rest and a body in motion continues with the same motion in a straight line, until acted upon by an external force.*

[ Nature prefers stability in the order of things ( stable equilibrium and minimum potential energy and when excess energy is available , it endeavours to dispense or distribute it with the available paths of least resistance ). Once the storm has passed , or activating energy bursts are no longer available , those bodies that are static are better off at rest eg. stones , boulders, petrified rocks and mountains; since the energy and forces required to kickstart them from rest and overcome their inertial resistance may be of a very high magnitude. But when the energy or the prime mover force is available ,the objects at rest are launched into an appropriate state of motion- not only for objects on the earth, but also man-made rockets, satellites and celestial objects such as comets, meteors, planets, earthquakes and volcanoes , suns, galaxies etc.

Once these billions of stellar systems are already pursuing their celestial motions in the vacuums of vast voids of space,(maintained and sustained after their violent and energetic beginnings),the course of nature ‘in progress’ is better off this way, rather than all these vast gigantic cosmic systems coming to a grinding and devastating halt or rest-which would involve astronomic forces as unimaginable as the forces which kicked them into motion in the first place! ]

**[ B ] State of rest and motion of a body**

Absence of any force acting on a body ensures status quo of its state of rest or motion and inertia is the property of a physical body that enables it to resist any change in its state of rest or motion ( Inertia of a body is the measure of its mass or the amount of matter in it . The less the mass of a body , the more easy it is for an applied Force to change its state of rest or motion , or mathematically , the greater is the acceleration ‘a’ produced . And we know this acceleration is a = F/ m , borrowing from the second law of motion , which is closely linked with the first law of motion ). This is clearly defined in **Newton’s first law of motion** and **Galileo’s law of Inertia**.

**Newton’s second law of motion** specifies how much or the quantity of change in the state of rest or motion of a body is produced by an external force acting on it.

Once the inertial resisting force of the body is overcome , the body fails to maintain its state of rest and accelerates and moves with a final velocity according to the second law of motion –

F x t = m x v or F. dt = m. dv ( **ie. F = m x a** )

What are all the processes that come into play from the instant the force is applied on the body with mass m initially at rest , till the expected change in its state of rest or motion is effected ? Is there a grey zone between the two laws of motion ? When a force is applied to the body at rest and the body apparently continues to be at rest, is it really so? Based on the second law of motion, it is imperative that the body should be in motion, maybe even very slowly , (with a velocity in mm/s or μm / s or acceleration in μm /s2) , not perceptible to our senses since frictional forces may be masking such a minute velocity and acceleration.

Experimentally this could be studied in ideal and vacuum conditions – eliminating contact friction by using ideal frictionless surfaces and any air drag friction in a vacuum chamber.

Here , the author seeks to analyze the various kinds of possible outcomes , when a body is subject to an impressed force , from its initial state of rest with maximum inertial resistance to an almost instantaneous change in its state of rest with minimum inertia effect.

1. **State of rest** If a body is static , either no force has been applied to it to dislodge or move it , or the force applied ( the impressed force ) was insufficient for moving it – ie. overcoming its resisting inertial force due to its inertial mass and other opposing forces like frictional forces etc.
2. **State of motion** A force of sufficient magnitude and duration can force the body at rest into a state of motion once the opposing force of inertia due to its mass is overcome.

ex. A push force ‘F ’ acting for a duration of time ‘t’ on a body of mass ‘m’ , overcomes its inertial resisting force and imparts to it a velocity ‘ v’ from a state of rest ‘v = 0’

Once the body is moving with a constant velocity ‘v’ , assuming no friction, air resistance or retarding forces are present , the inertia of the body helps in maintaining this constant state of motion ( mass times velocity or ‘mv’ ) . However its inertial force of resisting is still dormant and resident in the body , though inactive now.

The mass ‘m’ moving with a velocity ‘v’ again manifests its inertial force when it hits an obstacle , collides with another body or a push force ‘-F’ acts on it and seeks to bring it to rest once again in a time ‘t’ (eg. catching a ball moving at some velocity, and bringing its motion to a halt).

Or, when another force F ’ acts on a body to change its velocity from v to a new state of motion v’ ( eg. a bat impacting a ball and changing its velocity – cricketers , baseball players and sportsmen very often encounter these inertial forces in the course of their sports and games ).

**[C]**

**Inertia model,**

Regarding inertia, Newton said “A body exerts this force only when another force impressed upon it endeavours to change its condition ”

It is the capacity of a body to oppose or withstand the effect of a force impressed on it , thereby maintaining its present state of rest or motion.

This dormant inactive force or vis inertiae, and opposing capacity of innate force of matter or vis insita , to any change in its state of rest or motion is called inertial force.

**Physics forum website** and other Internet sources give these definitions of inertial force.

*An inertial force is a force that resists a change in velocity of a body.*

*It is equal to and opposite to , and in the opposite direction of an applied force , as well as a resisting force , based on Newton’s Action-Reaction law , law of Inertia and Newton’s second law of motion.*

*Mass of a body is a proportional measure of its inertia.*

*The Amount of inertia a body possesses is proportional to its mass. ( However inertia is not the same thing as mass or momentum ) . Amount of inertia here , is not the inertial mass or amount of matter in the body but a resisting force of a body to an impressed force on it, that seeks to prevent a change in its state of rest or motion and preserve its present state of rest or motion .*

Inertial force action opposes the applied force and delays its effect. Without any resisting inertial force , a body would be accelerated as per Newton’s second law of motion F= m.a ie. it would experience an acceleration during the duration t of the force equal to F/ m , where m is its inertial mass. Its final velocity would be v= a x t. With a resisting inertial force acting, there would be a delayed action of the force resulting in a decrease in its acceleration and its final velocity.

The **‘ Inertia effect ’** of a body is the manifestation of its inertial force at work , which resists the immeadiate action of an applied force to impart acceleration and change in velocity to it , or slows down its effect by resisting or blocking the action of the impressed force , by virtue of its inertial mass – an innate property of the bulk material of the body.

To find **FIR ,** the inertial resisting force

For this study, we make certain assumptions .

1. We assume that there is no other force impressed or applied on the body of mass m which is initially at rest on an ideal horizontal plane , besides a solitary finite non-zero value push force – no contact friction forces , no air-drag viscous forces etc. act on it. Moreover it is not a continuous force and the push force acts for a finite non-zero time t. Hence, under these ideal conditions , the body experiences a constant acceleration exactly equal to (F /m ) for the duration of time t , during which the force is applied to the body. Also the final velocity attained by the body v = a x t ( m/s ) , at the end of the push force acting on it.
2. The push force impressed on the body is assumed to be an ideal unit force pulse of magnitude F= 1 Newton, acting for t= 1 second. As shown in the figure , the push force pulse is an ideal one with a square shape and zero rise time ( unlike practical push forces which may be trapezoidal or exponential etc. ).

**F = 1 Newton**

**t = 1 second**

**Fig. 2 Ideal push force impressed on the body**

But even in these ideal conditions , the final velocity of the body will not be v = a x t but will vary from the maximum velocity of v m/s [where v = (F / m) x t] , depending on how much inertial resisting force is offered by the body to the impressed force.

1. By conducting this experiment under vacuum and ideal conditions ( eliminating any other competing forces including contact and non-contact frictional forces ) , the final velocities imparted to bodies subject to the same ideal push force F can be accurately and precisely measured. Can this slew of take-off velocities be a useful estimate of the inertial resistance offered by the concerned body ?
2. **Applied force Slope effect**

For simplification , the gradient of the applied force is assumed to be linear so that χ = dF / dt = F/ t

The gradient of the applied force slows down or increases the rate of overcoming of inertia of the body ( slow acting and fast acting forces relatively speaking ).

eg. when the gradient of the force χ = 100, 10, 31/2, 1 , 3-1/2,1/10 , 1/ 100 ….

~slope angle θ in degrees = 89, 84 , 60, 45, 30 , 5 , 0.5 ….

Also,for the purpose of simplifying our analysis , we assume an ideal push force with a vertical rising edge ie. χ = dF/dt , with infinite slope .

When inertial resisting forces are actively opposing the applied force F , the slope of the ideal applied force with vertical rising edge , is decreased or attenuated to ∞ > χ >=0. As χ decreases, the acceleration and velocity of the body also proportionately decreases and the attenuated force response now becomes F’ = χ .t

When the force F is fixed , and the mass of the body varied , the force response profile may vary from the extremes – vertical ( dF/dt = infinity when F >> FIR of m and v= 1 m/s ) and intermediate values eg. dF/dt = 1 ( v= 0.5 m/s ) , and dF/ dt = 0 ( v= 0 m/s , the body continues at rest since F << FIR of m ).

If the mass of the body is fixed and the force F is increased or decreased, the gradient dF/dt varies accordingly and the inertial resisting force is overcome sooner or later – ie. delay time in rise of F is less or more proportionately , and the final output velocity increases or decreases relatively in like manner.

1. **The inertial resisting force** offered by a body may be represented by a **Coefficient or a multiplier figure** , based on its **final velocity**.or alternatively on the amount of loss of final velocity due to inertial force effects.

eg. C = 0 , when the final velocity = 1 m/s ( no inertial resisting force apparently – the inertial force is overcome instantaneously since F>> FIR and m )

C = 0.5 when the final velocity = 0.5 m/s ( 50 % loss of velocity due to FIR )

C = 0.05 when the final velocity = 0.95 m/s ( 5 % loss of velocity due to FIR )

C= 0.95 when the final velocity = 0.05 m/s ( 95 % loss of velocity due to FIR )

1. Further , this **coefficient C may also be correlated with the amount of delay time** incurred in the response of the body to the applied force due to the resisting inertial force ; or the force profile response of the body ( examples a, b, x, y, z force response profiles shown in the Figure 3 )

In a , C = 0 and delay time = 0 s

In x , C = 0.1 and delay time = 0.1s ( after which FIR = 0 , ie. it is overcome )

In y , C = 1.0 and delay time = 1.0 s ( FIR is just overcome by F at t = 1 s )

In z , C = 10 and delay time = 10 s (only 5 % of FIR is overcome by F at t = 1s

- it would require 10 s of F to overcome the resisting inertial force )

1. In the absence of this experiment , we could analyze a few hypothetical cases of an ideal push force having normalized value 1 Newton , acting for 1 second on a unit mass of 1 Kg., acting under assumed ideal conditions. We can thus estimate the inertial resisting force offered by the 1 Kg. mass based on its final take-off velocity when the force of 1 N acts on it for 1 s.

Shown below are a few instances of force responses of a body when a push force F = I Newton acts for 1 second on a body with mass m Kg. ( velocities are computed using Newton’s second law of motion )

F = 1 N v=0.05 m/s ( final velocity v = 0.95 m/s , C= 0.1, m =1 Kg, χ =10)

1. (x) (y) (v=0.5 m/s , C= 1.0, m = 1 Kg , χ =1 )

(v=1 m/s, C= 0, F >>m , dF/dt = χ =∞ )

(z) (v=0.05 m/s , C= 10, m =1 Kg, χ = 0.1 )

(b) (v= 0 m/s , C>>, F << m , χ = 0 )

**T=1s Fig. 3 Shows the instantaneous response and delayed attenuated instances of force response profiles .( The Units of** χ is Ns-1 and of C is sKg-1 )

In the force response profile (x), time constant τ = 0.1s, from τ [s] = mC [ Kg s Kg-1 ] ,

for the force response profile (y) time constant τ = 1s ,

for (z) time constant τ = 10s. ,

for (a) τ = 0 , and for (b) τ = very large value

Interestingly, in (y) ,the time constant τ =1s ,from τ = mC =1[Kg] x 1.0 [s][Kg]-1, when m =1Kg.

If m is now increased to m= 10 Kg ,for a body made of the same material and having the same time constant per unit mass C= 1.0 [s][Kg]-1 ,then τ = 10s , v= .005 m/s and the force response profile slips down to (z) ( where t = 10s for m= 1 Kg and C=10 [s][Kg]-1 )

If m is now increased to m= 100 Kg , and the body is made of the same material or same C , τ = 100s, v= 0.5x10-4 m/s, the force response slips further down.

If m= 1000 Kg., τ = 1000s , v = 0.5x10-6 m/s or half a micron per second !

Eventually , as m keeps increasing , F<< m , v is infinitesimally small , and we finally reach (b) the static state of rest force response profile , where inertial resisting force overcomes the force F and the body continues to be at rest !

The **inertial resisting force of a body , FIR**, due to its incumbent inertia , is proportional to its mass ‘m’. If FIR is equivalent to a fraction of the mass of the body, can it be quantitatively estimated ? The mismatch in Units of force and mass needs to be addressed - the lack of dimensional integrity in their relation , which will be shown later.

We can say FIR is proportional to the mass m of the body, and C, the time delay of the rise of force F based on mC - the mechanical time constant of the system with mass m.

Let us say initially, **FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1 ....................... (Equation 1). Here C, a Coefficient or multiplier of the inertial mass of the body is the time-constant per unit mass of a mass-force system,with the units of s/Kg and τ[s] = mC [Kg][s][Kg]-1 is the mechanical time-constant of the mass-force system . The factor C controls the delay time of the rising edge of the push force applied to the body and needs to be estimated if not accurately determined (akin to the first order response of a mechanical system ) : k is not really a constant but varies linearly from – χ to 0 [N /s] and causes the FIR to vary from –F Newtons ( when the force F is applied ) to 0 Newtons , when the inertial resisting force is overcome.

It can be finally shown that **FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1,where **k = [χ ( 1-n) / n]** and t= mc /n is a fraction of the time constant and n can take real values n >= 1 to n 🡪 ∞ ( the limiting fraction when n 🡪 ∞ corresponds to t=0 , when the force F is applied ) .

That is , **FIR** [N] = **[χ ( 1-n) /n] mC** [N][s]-1[Kg][s]Kg]-1 ……… …............(Equation 2)

Numerically & graphically, it can be shown that **FIR** [N] = **(χ t – F)**[N]…..…(Equation3)

From equations 1 and 3 , we can show that **k** [N][s]-1= **(χ t – F) / mC** [N][s]-1, where χ is the rate of rise of the applied force F in the presence of a resisting inertial force and this resultant force **F’=χ. t** [N][s]-1[s] . For simplification, we assume a linear rise of force and assume one mechanical time constant during which the inertial resisting force opposes the applied force F , which was earlier assumed to be an ideal pulse with a vertical rising edge .

If we consider this to be a natural system which is logarithmic in response, then we could consider 5 time constants before the final value of the rising force almost equals the actual value of the applied force. In which case , modeling the bodys’ inertial force and its exponential response can be conveniently done using a RC circuit.

The mechanical time constant of this mass-force system is τ = m C . In this equation the mass of the body m is equivalent to a resistance opposing the flow of current ( acceleration a , in this case ) where a = F/ m according to Newton’s second law of motion analogous to I= V/ R in an electrical Ohmic circuit .

C contributes to the time delay in the system. It is the time constant per unit mass – ie. the capacity of the body with unit mass to resist an imposed force F , and delay its rise .

( C would vary for unit masses of different material compositions eg. metals, non-metals, flexible , rigid materials etc. )

It can be modeled as a capacitor , since it results in a time delay and delays the effect of the applied force on the build-up of velocity and acceleration of the body – the bodys’ inertia is its capacitive slug to slow down the immeadiate action of an impressed force into a changed state of rest or motion as per Newton’s laws of motion and it is manifest as a resisting force.

Summing the earlier points enumerated –

Inertia of a body is a measure of its mass or the amount of matter in the body. ( inertial mass , SI units Kg. )

Inertial resisting force offered by a body is not its inertia , but proportional to its inertia or inertial mass.

The inertial resisting force of a body is thought to be proportional to its mass but not equal to its mass ; and it is a force ( SI units of Newton ) . Hence the inertial resisting force of a body produced by a force acting on a body with mass m , can be expressed as a numerical quantity equal to its mass times a Multiplier or coefficient C.

C is the capacity of a body to delay the action of a force applied on it and it may be defined as 0 < C < 1

However it is not possible to find the exact value of this coefficient C which would need a complete knowledge of the atomic structure and physical characteristics of the body , quantum forces and quantum behavior ; all the details of its micro and macro structure and behavior.

Hence an indirect estimate or approximate figure of the inertial resistance offered by the body can be obtained by knowledge of the time delay in the rise of its resultant force response or the amount of slowing down of the action of the impressed force as measured by the reduced rate of growth of acceleration and final velocity reached of the body.

The final velocity attained by the body could be measured experimentally under ideal, vacuum conditions ( using near perfect horizontal and frictionless contact surfaces and effectively eliminating air drag etc. which would otherwise mask the inertial forces required to be experimentally estimated ).

Once the final velocity of the body is accurately measured , its force response profile can be graphically or numerically obtained . That is the slope of the resultant force χ is known and also the time delay before the FIR is overcome which is the mechanical time constant of this mass-force system τ = m C .

Knowing χ and m C , the FIR at different time instants can be estimated from the equations earlier derived for FIR –

Equation 1 **, FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1

Equation 2 **, FIR** [N] = **[χ ( 1-n) /n] mC** [N][s]-1[Kg][s]Kg]-1 where **t = mC/ n**

& Equation 3 , **FIR** [N] = **(χ t – F)**[N]

**Are there any Relativistic effects on inertial resisting forces ?**

If a body of mass m (eg. a particle ), is accelerated to very high speeds , it experiences a mass dilation as per Einstein’s Special theory of relativity to **m**/ **√ ( 1 – v2/c2 ) ,** where m is the inertial rest mass of the body , c is the velocity of light and v is the velocity of the body .

Hence the body would now offer a proportionately much higher inertial resisting force to a change in its state of motion. The increase in the inertial reactive resisting force, FIR = **kmC** , being proportional to the augmented mass of the body with the Relativistic correction of mass dilation, example when this highly accelerated body is suddenly brought to a halt.( assuming that **C** , the characteristic time constant per unit mass of the body is constant , which may not be really true in the heavily accelerated dilated body ).

Recall the Equation 1 , **FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1  which should be valid , but

with the Relativistic correction **FIR** [N] = **kC m** / **√ ( 1 – v2/c2 )** [N][s]-1[Kg][s]Kg]-1 …Equation 4

However, the change in the **FIR** gradient due to mass variations , would be more complex, compounded by changes in the characteristic decay gradient expected, as compared to the linear variations in k from – χ to 0 in time **t** = **mC ,** when the body was initially impelled into motion .

The new time constant of decay **mC ,** varieswith time and cannot be assumed to be linear **,** since itis now **mC** / **√ (1 – v2/c2 )** which wouldvary non-linearly as v varies non-linearly during the stopping time duration. Also different variations in k , the gradient of decay are expected.

Alternatively , the variations in mass due to inverse dilation during the stopping phase , could be appended to the constant k . The new k is now k / **√ (1 – v2/c2 )** which defines the new decaying gradient of the **FIR**  as it decays to zero during the time **t** = **mC .**

**Are these equations also linked to Gravitational force ?**

A new viewpoint of Newton’s Universal law of Gravitation

As per Newton’s universal law of Gravitation ,the force of attraction between any two bodies is proportional to the masses of the bodies and inversely as the square of the distance between the masses.

Eg. **FG = G M earth X m moon / r 2** , where G is the Universal Gravitational constant and r is the distance between the earth and the moon.

Relative to the earth , the moon is orbiting the earth , and at each point on its orbit , the moon experiences a centripetal acceleration g = G M earth **/**  r 2 , which causes a limited free fall , before its tangential motion whisks it into its next contiguous position on its orbit around the earth.

As far as the centripetal gravitational force and acceleration of the moon towards the earth is concerned , it is dropping from rest earthwards at a known rate based on the value of FG experienced by the moon at that orbital distance . .

Hence the inertial resisting force offered by the moon based on the earlier derived equation ,

Equation 1 **, FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1

with respect to the motion of the moon , must come into the picture -

ie. **FIR moon = k moon X m moon X C moon**  ( where **C moon** is the time constant per unit mass of the moon and k is the pseudo-constant that defines the rate at which the inertial resisting force of the moon is being overcome ( k = **[χ ( 1-n) /n]** [N][s]-1 and **t = m moon C moon / n ,** and nis a fraction of the time constant and n can take real values n >= 1 to n 🡪 ∞ )

Hence a small correction to the value of FG  cannot be ruled out , based on its inertial resisting force .

And , **FG = G Mearth X mmoon / r 2** – **kmoon X m moon X C moon**  ………………… Equation 5

Similarly , the attractive force experienced by the earth due to the moon is

**FG = G m moon X M earth / r 2 –** **k earth X M earth X C earth**  ……………….Equation 6

Equating the two Force quantities in equations 5 and 6 ,

**k moon X m moon X C moon**  = **k earth X M earth X C earth**  ………………….Equation 7

Thus **m moon / M earth  = k earth X C earth** / **k moon X C moon** ………………….Equation 8

Since the constituent atoms are similar on the earth and the moon ; let us assume that C **moon ,** the time constant per unit mass of the moon is equal to C **earth ,** the time constant per unit mass of the earth ( which mayactually vary in practice ). Hence ,

**m moon / M earth  = k earth**  / **k moon**  ………………………………………………..Equation 9

ie. the rate at which inertial resisting force of the moon or earth body is being overcome is inversely proportional to its mass , which seems logical .

When we consider the modern Gravitational equation , modification of Newtons Universal law of Gravitation -

**FG = G M earth X m moon / r ,** still the above equation 7, 8 and 9 should hold true .

**[D]**

**Conclusion**

In conclusion , the law of Inertia tells us how the state of rest or motion of a body is maintained and Newton’s second law of motion defines how the state of rest or motion of a body is changed by a force.

In this article , the author has strived to understand the inner mechanisms of inertial resisting force in a body mass-force system and obtain estimates of the inertial resisting forces that come into play and how they are overcome.

Many assumptions have been made as shown earlier , and simplification of the system has been adopted in order to simplify the analyses and estimation of inertial resisting forces eg. the systems are assumed to be linear , with a linear rise of force F , and consequent linear build-up of acceleration and velocity , and only one mechanical time constant is assumed.

In practical reality it is not so simple , and being a natural system where logarithmic and exponential behavior is expected, there would be an exponential build-up of the force response profile ( like a capacitor charging ) and the acceleration and velocity variations would also be non-linear and you would have to consider 5 to 10 mechanical time constants . Such a model is more complex and is not discussed here.

Practical experimental set-ups may be designed as future continuation of this work – such as motion studies in a vacuum chamber with frictionless contact surfaces.

Secondly , phantom studies can be done with selected material blocks and also phantom blocks having implanted acceleration or other sensors in a 3D matrix of desired resolution and suitable electronics processing to collect real-time data and collate it, when the sensor phantom blocks are subjected to a specified Force. The sensors can be with wired outputs or attached to a wireless module piggy-back on the same block.

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**References -**

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**Additional notes**

Inertial force

**What it is**

An inertial force is a force that resists a change in velocity of a body .

It is equal to and opposite to , in the opposite direction of an applied force , as well as a resisting force , based on Newton’s Action-Reaction law and the law of Inertia or Newton’s 1st law of motion and Newton’s II nd law of motion.

**What it does**

Inertial force action passively opposes the applied force and delays its effect.

Without any Inertial force , acceleration of a body would be instantaneous from rest and equal to F/m as per Newtons 2nd law of motion and the change in velocity of the body produced depends on the time of action of the applied force . ( v = aX t )

Inertial resisting forces of a body decrease its rate of growth of acceleration from rest , its final velocity attained and the slope of the resultant force acting on it. ie. it delays the action of the applied force.

**What exactly is it ? How is it estimated ?**

Inertia of a body is a measure of its mass or the amount of matter in the body. ( inertial mass , SI units Kg. )

Inertial resisting force offered by a body is not its inertia , but proportional to its inertia or inertial mass.

The inertial resisting force of a body is thought to be proportional to its mass but not equal to its mass ; and it is a force ( SI units of Newton ) . Hence the inertial resisting force of a body produced by a force acting on a body with mass m , can be expressed as a numerical quantity equal to its mass times a Multiplier or coefficient C.

C is the capacity of a body to delay the action of a force applied on it and it may be defined as 0 < C < 1

However it is not possible to find the exact value of this coefficient C which would need a complete knowledge of the atomic structure and physical characteristics of the body , quantum forces and quantum behavior ; all the details of its micro and macro structure and behavior.

Hence an indirect estimate or approximate figure of the inertial resistance offered by the body can be obtained by knowledge of the time delay in the rise of its resultant force response or the amount of slowing down of the action of the impressed force as measured by the reduced rate of growth of acceleration and final velocity reached of the body.

The final velocity attained by the body could be measured experimentally under ideal, vacuum conditions ( using near perfect horizontal and frictionless contact surfaces and effectively eliminating air drag etc. which would otherwise mask the inertial forces required to be experimentally estimated ).

Once the final velocity of the body is accurately measured , its force response profile can be graphically or numerically obtained . That is the slope of the resultant force χ is known and also the time delay before the FIR is overcome which is the mechanical time constant of this mass-force system τ = m C .

Knowing χ and m C , the FIR at different time instants can be estimated from the equations earlier derived for FIR –

Equation 1 **, FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1

Equation 2 **, FIR** [N] = **[χ ( 1-n) /n] mC** [N][s]-1[Kg][s]Kg]-1 where **t = mC/ n**

& Equation 3 , **FIR** [N] = **(χ t – F**)[N]

**Are there any Relativistic effects on inertial resisting forces ?**

If a body of mass m is accelerated to very high speeds , it experiences a mass dilation as per Einstein’s Special theory of relativity to **m**/ **√ ( 1 – v2/c2 ) ,** where m is the inertial rest mass of the body.

Hence the body would now offer a proportionately much higher inertial resisting force to a change in its state of motion. The increase in the inertial reactive resisting force, FIR = **kmC** , being proportional to the augmented mass of the body with the Relativistic correction of mass dilation, example when this highly accelerated body is suddenly brought to a halt.( assuming that **C** , the characteristic time constant per unit mass of the body is constant , which may not be really true in the heavily accelerated dilated body )

Recall the Equation 1 , **FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1  which should be valid , but

with the Relativistic correction **FIR** [N] = **kC m** /**√ ( 1 – v2/c2 )** [N][s]-1[Kg][s]Kg]-1

However, the change in the **FIR** gradient due to mass variations , would be more complex, compounded by changes in the characteristic decay gradient expected, as compared to the linear variations in k from – χ to 0 in one time constant time **t** = **mC ,** when the body was initially impelled into motion .

The new time constant of decay **mC ,** varieswith time and cannot be assumed to be linear **,** since itis now **mC** / **√ (1 – v2/c2 )** which wouldvary non-linearly as v varies non-linearly during the stopping time duration. Also different variations in k , the gradient of decay are expected.

Alternatively , the variations in mass due to inverse dilation during the stopping phase , could be appended to the constant k . The new k is now k / **√ (1 – v2/c2 )** which defines the new decaying gradient of the **FIR**  as it decays to zero during the time **t** = **mC .**

**Are these equations also linked to Gravitational force ?**

A new viewpoint of Newton’s Universal law of Gravitation

As per Newton’s universal law of Gravitation ,the force of attraction between any two bodies is proportional to the masses of the bodies and inversely as the square of the distance between the masses.

Eg. **FG = G M earth X m moon / r 2** , where G is the Universal Gravitational constant and r is the distance between the earth and the moon.

Relative to the earth , the moon is orbiting the earth , and at each point on its orbit , the moon experiences a centripetal acceleration g = G M earth **/**  r 2 , which causes a limited free fall , before its tangential motion whisks it into its next contiguous position on its orbit around the earth.

As far as the centripetal gravitational force and acceleration of the moon towards the earth is concerned , it is dropping from rest earthwards at a known rate based on the value of FG experienced by the moon at that orbital distance . .

Hence the inertial resisting force offered by the moon based on the earlier derived equation ,

Equation 1 **, FIR** [N] = **kmC** [N][s]-1[Kg][s]Kg]-1

with respect to the motion of the moon , must come into the picture -

ie. **FIR moon = k moon X m moon X C moon**  ( where **C moon** is the time constant per unit mass of the moon and k is the pseudo-constant that defines the rate at which the inertial resisting force of the moon is being overcome ( k = **[χ ( 1-n) /n]** [N][s]-1 and **t = m moon C moon / n ,** and nis a fraction of the time constant and n can take real values n >= 1 to n 🡪 ∞ )

Hence a small correction to the value of FG  cannot be ruled out , based on its inertial resisting force .

And , FG = G Mearth X mmoon **/**  r 2 – **kmoon X m moon X C moon**  ………………… Equation 5

Similarly , the attractive force experienced by the earth due to the moon is

**FG = G m moon X M earth / r 2 –** **k earth X M earth X C earth**  ……………….Equation 6

Equating the two Force quantities in equations 5 and 6 ,

**k moon X m moon X C moon**  = **k earth X M earth X C earth**  ………………….Equation 7

Thus **m moon / M earth  = k earth X C earth** / **k moon X C moon** ………………….Equation 8

Since the constituent atoms are similar on the earth and the moon ; let us assume that C **moon ,** the time constant per unit mass of the moon is equal to C **earth ,** the time constant per unit mass of the earth , which mayactually vary in practice.

**m moon / M earth  = k earth**  / **k moon**  ………………………………………………..Equation 9

ie. the rate at which inertial resisting force of the moon or earth body is being overcome is inversely proportional to its mass , which seems logical .

When we consider the modern Gravitational equation

**FG = G M earth X m moon / r ,** still the above equation 7, 8 and 9 should hold true .

0.5

0.25

1s

+1N

F

F ‘ for Χ = 1

F’

Avg.‘a’

Inst. Vel.

Avg. vel

0.5 m/s2 . Rev. Avg. vel

0.5 m/s Rev. Inst. vel

+ 0.5N

. Rev Avg.‘a’

FIR

+0.25N

+0.125N

0 N,0 s

0.125s

-0.125N

Time t ( sec)

-0.25N

-0.5 m/s2

-0.5N

**Diagram shows** the effects of a force of 1N applied on a mass of 1 Kg. and the resultant force F’

produced on the body for Χ = 1, with the expected average forward acceleration produced and changes in

instantaneous velocity and acceleration .( As also the average reverse virtual ? acceleration that corresponds to the

inertial resisting force FIR and the reverse profile of acceleration and velocity during the one second when the force

was applied. .( These are based on an intuitive experiment and not real experimental data )

-F

-1.0N

**Diagram shows** the effects of a force of 1N applied on a mass of 1 Kg. and the resultant force F’

produced on the body for Χ = 1, with the expected average forward acceleration produced and changes in instantaneous velocity and acceleration .( As also the average reverse or retro virtual acceleration that corresponds to the inertial resisting force FIR and the reverse profile of acceleration and velocity during the one second when the force was applied. .( These are based on an intuitive experiment and not real experimental data )