Physics of Yanks and Jerks & Tugs and Snaps…

It was Aristotle who first stated that ' *All things are in motion* ', with reference to the physical universe, and he sought to *quantify matter and motion and to understand the cause of motion.*

Galileo, by his great genius, laid to rest philosophical speculations about the cause of motion. Instead he theorized that it is *sufficient to study and determine what causes a change in motion first* and then you will know what is the cause of motion in the universe . He conducted experiments with inclined planes and found experimentally that a force is required to produce a change in motion of a body. Galileo also first stated the ***' law of Inertia '*** which was later clearly defined and developed by the scientific genius of Newton into the ***First law of motion***.

Newtons' ***Second law of motion*** gives us the equation that relates the amount of change in motion of a body to the amount of force applied on the body.

$F = m \times a$ or $F = m \times dv/dt = m \times d/dt\,(v)$ or the ***rate of change of momentum*** of a body is ***directly proportional to the applied force*** and takes place in the direction of the applied force (ie. the ROM or rate of change of momentum of a body is directly proportional to the applied force). And we know the first law of motion is also a natural outcome of the second law of motion – ie. when the applied force is zero , then dv/dt is also zero , hence there is no change in the velocity of the body and its state of rest or motion is maintained.

On Yanks and Jerks

But what if the force applied on the body itself is varying with time ? Then the instantaneous values of force applied produce instantaneous changes in dv/dt or acceleration of the body.

So we see here a natural extension that follows from Newtons' second law of motion, $\mathbf{F = m \times a}$

Differentiating Newtons' classical equation of his 2nd law of motion gives the equation $\mathbf{F' = m \times a'} = m \times da/dt$ or $dF/dt = m \times d^2v/dt^2$ or $dF/dt = m \times d/dt(dv/dt) = d/dt\,(d/dt\,(mv))$

ie. the rate of change of the rate of change of momentum of a body is directly proportional to the rate of change of the force applied to it and takes place in the instantaneous direction of the derivative of the force applied to it (ie. the ROROM of a body is directly proportional to the ROF, or rate of change of the force applied). So a sudden increase in a force applied to a body in a short time, followed by a sudden decrease in the applied force produces a sudden increase in the momentum of the body followed by a sudden decrease in its momentum or a sharp rise in acceleration of the body followed by a sharp fall in acceleration or deceleration.

The force build-up and its decline could be linear or non-linear .For simplication , we can assume a linear growth and decay of the force in a small time interval , so that the rise and fall of acceleration produced in the body is also linear.

As suggested on USENET, the assigned term for F' or dF/dt is **Yank** ,**Y**, and the rate of change of acceleration is denoted by the term **Jerk** , **j** . **Yank = mass x jerk**

Jerk $j = da/dt = d^2v/dt^2 = d^3x/dt^3$. Hence Jerk is also called the 3rd derivative of motion of a body, and it represents a sudden jolt or sudden increase in acceleration of a body (eg. a bike or car etc.) from rest or from a steady acceleration , or a sudden deceleration of a body to rest or a constant acceleration.

'Jerk ' in a system , occurs due to a change in the force applied to it.

The application of force does not happen instantly nor is acceleration instantaneous. Just as the applied force builds-up during a time period from zero to its final value , so also the acceleration picks up from zero to its maximum value during the same time interval. ***Jerk is the change in acceleration experienced by the body over time .***
(The force build-up and its decline could be linear or non-linear. For simplication, we assume a linear growth and decay of the force in a small time interval , so that the rise and fall of acceleration produced in the body is also linear).

An inexperienced driver of a vehicle applies brakes (increased braking force) suddenly, in a shorter time, and this higher Yank force causes faster deceleration or higher jerk of the vehicle - which is unpleasant to his passengers who experience a severe jolt ! Whereas an experienced driver gradually applies the brakes (ie. a lesser Yank or rate of changing force), causing slow deceleration or jerk and less jolt to his passengers.

Similarly, a good driver accelerates the vehicle more smoothly from rest over a longer time duration (ie. lessYank which produces less Jerk), but a poor driver 'races' or pumps the accelerator pedal more and this higher Yank causes a higher jerk or faster increase in acceleration of the vehicle in a shorter time, so his passengers experience a more 'jerky' ride and not a smooth one.

(A steady position of the acceleration pedal is equivalent to no change in force applied , hence acceleration is zero and the vehicle cruises with a steady speed giving the passengers a comfortable ride)

Measuring the third derivative of motion or jerk is important in Aerospace engineering and a jerkmeter is used for this purpose.

We know from Newtons' 2nd law of motion applied in Electrical science, that the force on a conduction electron in an electrical wire ,due to an applied Electric field applied across the wire is F = e x E = m x a

In Electrical engineering, when a steady dc voltage (electric force) is applied across an electrical wire, the steady dc voltage results in a steady electric field in an electrical wire which results in a constant acceleration on the electrons which translates to a constant electron drift velocity during the mean free time between collisions of electrons with the lattice ions. A slow and steady rate of voltage applied to the same wire, produces a proportional slow increase in the acceleration or slow jerk of the conduction electrons and subsequent steady rise in its drift velocity .

But when a dc voltage surge occurs, there is a sudden increase in voltage from its steady state value like a sudden rise in Yank force on a physical body. During a voltage surge, the sharply increasing voltage produces a fast increasing electric field in the wire , which results in a rapid increase in the acceleration or a fast jerk of conduction electrons which increases sharply their drift velocity producing a high surge current more than the steady dc current it was carrying before the voltage surge appeared.

Of Tugs and Snaps or Jounce

If the applied force on a body is constant and unchanging with time, we know from the second law of motion **F= m x a**, or that the body experiences a constant acceleration a = F / m , also called the second derivative of motion d^2x/dt^2 .

If the applied force varies linearly with time like a rising ramp signal, F = k x t ,
then the **Yank, Y** = dF/dt = k and the **Jerk, j** = da/dt = d^3x/dt^3 , the third derivative of motion.

Also from the second law of motion , F = m x a = k x t , so Yank= dF/dt = k = m x da/dt , and **Yank = mass x jerk** & da/dt = k/m, a constant rate in this case.
or **Y = m x j ,** from the derivative of Newtons' second law of motion F = m x a.

If the applied force is of an exponential nature, F= e^{kt} , Y = dF/dt = k x e^{kt} =k x F = m x da/dt. Again **Y = m x j** and da/dt = k F/ m , not a constant rate in this case, and varying as per the instantaneous values of force with time .

Tug (T) is the 2^{nd} derivative of force d^2F/dt^2 , and **Snap** (**s**), is the 4^{th} derivative of motion d^4x/dt^4 and the 2^{nd} derivative of acceleration d^2a/dt^2 .

Differentiating Newtons' classical equation of his 2^{nd} law of motion twice, gives the equation **F''= m x a''** = m x d^2a/dt^2 or d^2F/dt^2 = m x d^4x/dt^4

or d^2F/dt^2 = m x d/dt [d/dt(dv/dt)] = d/dt[d/dt (d/dt (mv))] ie. the rate of change of the rate of change of the rate of change of momentum of a body is directly proportional to the rate of change of the rate of change of the force applied to it and takes place in the instantaneous direction of the derivative of the derivative of the force applied to it (the ROROROM of a body is directly proportional to the ROROF, or the rate of change of the rate of change of the force applied)

Tug and **Snap** are obtained as shown above by double differentiation of the 2^{nd} law of motion **F''= m x a''** or $d^2F/dt^2 = m \times d^4x/dt^4$

ie. **Tug = mass x snap** , or **T = m x s**

If the force is a linearly increasing ramp F= k x t , then

Yank Y = dF/dt = k as shown earlier and

Tug T = $d^2F/dt^2 = dY/dt = 0$ ie. there is no Tug for a linear ramp force.

But if at some instant of time t , there is a surge or a sudden rise and fall of the force from its normal expected value , then the Yank dF/dt will not be constantly k , but vary during the surge time period in the neighbourhood of t.

The Tug values are zero when the force values are as per a linear ramp and Yank dF/dt is k ; but at the instant t when the surge force appears , the Yank is not a constant k and its values change suddenly as per the surge rise and fall, hence the Tug values are non-zero and take the form of a noise pulse or glitch from an otherwise uneventful straight line !

This could be useful in automated instrumentation positioning systems.

The Hubble space telescope uses the 4^{th} derivative of motion d^4x/dt^4, **Snap or Jounce** , and the second derivative of force d^2F/dt^2 **Tug** , in addition to the 3^{rd} derivative of motion jerk and the first derivative of force Yank , since its positioning accuracy is very critical for astronomical observations.

If the force considered is an exponential force $F = e^{kt}$,

Yank Y = $dF/dt = k\, e^{kt}$

Tug T = $d^2F/dt^2 = dY/dt = k^2 e^{kt}$

And if the force considered is a sinusoidal force F= ASinwt ,

Yank Y = dF/dt = AwCoswt , which varies co-sinusoidally as the sinusoidal force F and

Tug T = d^2F/dt^2 = dY/dt = Aw^2 Sinwt , which varies sinusoidally as the sinusoidal force F.

A surge or a sudden non-linearity in the applied force could be detected by the sharp deviations and sudden changes in the detected values of Tug & Yank and jerk & snap from their respective expected values.

Snatch and Crackle etc.

Snatch (S) is the 3rd derivative of force d^3F/dt^3 , and **Crackle (c)**, is the 5th derivative of motion d^5x/dt^5 and the 3rd derivative of acceleration d^3a/dt^3 .

Differentiating Newtons' classical equation of his 2nd law of motion thrice, gives the equation **F'''= m x a'''** = m x d^3a/dt^3 or d^3F/dt^3 = m x d^5x/dt^5

or d^3F/dt^3 = m x d/dt{d/dt [d/dt(dv/dt)]} = d/dt{d/dt[d/dt (d/dt (mv))]}

ie. the rate of change of the rate of change of the rate of change of momentum of a body is directly proportional to the rate of change of the rate of change of the rate of change of the force applied to it and takes place in the instantaneous direction of the derivative of the derivative of the derivative of the force applied to it (the ROROROROM of a body is directly proportional to the ROROROF, or the rate of change of the rate of change of the rate of change of the force applied).

Snatch = mass x crackle , or **S = m x c**

Snatch , Crackle and further derivatives of force and motion are more of academic interest and are not applied practically .

F''''= m x a'''' corresponds to **Shake = mass x pop or Sh = m x p**

F'''''= m x a''''' corresponds to **Lock = mass x drop or L = m x d**

Conclusion

Hence we could comprehensively state the Force changing scenario with the corresponding changes in momentum in the equation below which I suggest –

F'''''+ F''''+ F'''+ F'' + F' + F + 0 = 0 + mu + mdv/dt + md^2v/dt^2 + md^3v/dt^3 + md^4v/dt^4 + md^5v/dt^5 + + md^6v/dt^6

In the LHS of the above equation , Force = 0 produces no change in motion ie. the body continues in its initial state of rest , SOR or in its initial state of motion, SOM in accordance with Newtons' First law of motion. Also the initial state of momentum mu of the body is preserved since F = 0.

Subsequently , a force F applied on the body produces a rate of change of momentum of the body in accordance with Newtons' Second law of motion. Further derivatives of force F acting on the body produce corresponding changes in rate of derivatives of momentum as discussed earlier in this note and indicated in the equation shown above. As shown earlier , these are obtained as higher derivatives of the Second law of motion.

References

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